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Fundamental Algorithms 7 - Solution Examples

Exercise 1 (Hash Function)

Let $n = 1000$. Compute the values of the hash function $h(k) = \lfloor n(ak - \lfloor ak \rfloor) \rfloor$ for the keys $k \in \{61, 62, 63, 64, 65\}$, using $a = \frac{\sqrt{5}-1}{2}$. What do you observe?

Solution:

k	61	62	63	64	65
$h(k)$	700	318	936	554	172

The hash function is "non-smooth": similar entries lead to different hash values.

Exercise 2 (Hash Table)

Let T by a hash-table of size 9 with the hash function $h : U \to \{0, 1, \ldots, 8\}, k \mapsto k \bmod 9$. Write down the entries of T after the keys 5, 28, 19, 15, 20, 33, 12, 17, and 10 have been inserted. Use chaining to resolve collisions.

Solution:

i	0	1	2	3	4	5	6	7	8
$T[i]$	[]	[10,19,28]	[20]	[12]	[]	[5]	[33,15]	[]	[17]

The []-notation denotes the lists that are stored in each hash table slot.

Exercise 3 (Open Hash)

Now, let T be a hash table of size 11, using open addressing with the following hash functions

1. $h(k,i) := (k + i) \bmod 11$
2. $h(k,i) := \left(k \bmod 11 + 2i + i^2\right) \bmod 11$
3. $h(k,i) := (k \bmod 11 + i \cdot (k \bmod 7 + 1)) \bmod 11$

Insert the keys 5, 19, 27, 15, 30, 34, 26, 12, and 21 (in that order) and state which keys require the longest probe sequence in the resulting tables.

Solution:

1. Linear probing:

i	0	1	2	3	4	5	6	7	8	9	10
$T[i]$		34	12		15	5	27	26	19	30	21

   Longest probe sequence is 4 (for 26).

2. Quadratic probing:

i	0	1	2	3	4	5	6	7	8	9	10
$T[i]$	30	34	27		15	5		26	19	12	21

Longest probe sequence is 2 (for 27 and 12).

3. Double hashing probing:

i	0	1	2	3	4	5	6	7	8	9	10
$T[i]$	30	27	12	21	15	5		34	19		26

Largest probe sequences is 5 (for 34 and 21).

Note: Contrary to this example, double hashing usually beats linear or quadratic probing. Moreover, using a larger table for open addressing is recommended.

Exercise 4 (Hashing the Universe)

Consider a universe U of keys, where $|U| > mn$, and a hash function $h : U \to \{0, 1, \ldots, n-1\}$. Show that there are at least m elements of U which are mapped to the same hash value, i.e. there is a subset A of U with $|A| = m$ and $h(a_1) = h(a_2)$ for all $a_1, a_2 \in A$.

Solution:

Assume the opposite, i.e. that for all n values of the hash function the number of elements in U that are hashed to this value is smaller than m. As a consequence, the number of elements that are hashed to any of the n keys is smaller than nm. This contradicts the fact that U is considered to have more than nm elements. Hence, our assumption has to be false, and there has to be at least one subset containing at least m elements.